RIMS workshop “Nonlinear and Random Waves”
October 3-5, 2022, Kyoto, Japan

Program and Abstract

<u>Oct. 3 2022</u>

9:00-10:00 Andrea Nahmod, University of Massachusetts Amherst
“Gibbs measures and propagation of randomness under the flow of nonlinear dispersive PDE”

10:20-11:20 Mamoru Okamoto, Osaka University
“Stochastic quantization of the Φ_3^3 model”

11:20-13:50 Lunch

13:50-14:50 Itsuko Hashimoto, Kanazawa University
“Asymptotic behavior toward radially symmetric stationary solutions of the compressible Navier-Stokes equation”

15:00-15:30 (Short communication) Hirotatsu Nagoji, Kyoto University
“Renormalization of stochastic nonlinear heat and wave equations driven by subordinate cylindrical Brownian noises”

15:30-16:00 Coffee break

16:00-17:00 Laurent Thomann, Universite de Lorraine
“Almost sure scattering for the one dimensional nonlinear Schrödinger equation”

17:00-18:00 (HYD2 Joint meeting with RIMS) Gennady EI, Northumbria University
“Emergent hydrodynamics of soliton gases in integrable systems”

<u>Oct. 4 2022</u>

9:00-10:00 Sandra Cerrai, University of Maryland
“On the small mass limit for some stochastic wave equations with variable friction and polynomial nonlinearities”

10:10-11:10 Masaya Maeda, Chiba University
“Small energy stabilization of 1D nonlinear Klein-Gordon equation with potential”

11:20-12:20 Yukimi Goto, Kyushu University
“Born-Oppenheimer potential energy surfaces for Kohn-Sham models in the local density approximation

12:20-14:30 Lunch

14:30-15:30 Aurelien Deya, Universite de Lorraine
“A few results about the hyperbolic Anderson model”

15:30-16:00 Coffee break

16:00-17:00 Anne-Sophie de Suzzoni, Ecole Polytechnique

“General remarks on the propagation of chaos in wave turbulence and application to the incompressible Euler dynamics”

17:00-18:00 (HYD2 Joint meeting with RIMS) Christof Melcher, RWTH Aachen University
“Emergent spin-orbit coupling and rotating skyrmions”

Oct. 5 2022

9:30-10:30 Takiko Sasaki, Musashino University
“The combined effect of one space dimension beyond the general theory for nonlinear wave equations”

10:40-11:10 (Short communication) Minami Watanabe, Tsuda College
“Global dynamics of nonlinear Schrödinger equation with double power nonlinearity”

11:20-12:20 Svetlana Roudenko, Florida International University
“Dynamics of solutions in stochastic nonlinear Schrödinger equation: critical and supercritical cases”

Andrea Nahmod, University of Massachusetts Amherst
Title: Gibbs measures and propagation of randomness under the flow of nonlinear dispersive PDE.

Abstract: In groundbreaking work, Bourgain '96 put forward a random data theory to study the existence of strong solutions on the statistical ensemble of Gibbs measures associated with dispersive equations. Despite numerous follow-up works to those of Bourgain's, fundamental questions remained open. How does a given initial random data get transported by a nonlinear flow? If it is Gaussian initially, how does this Gaussianity propagate? What is the description of the solution beyond the linear evolution? In work joint with Yu Deng and Haitian Yue, we developed the theory of random tensors, a powerful new framework which allows us to unravel the propagation of randomness under the NLS flow beyond the linear evolution of random data, and answer these questions in an optimal range relative to what we define as the probabilistic scaling. In particular, we establish the invariance of Gibbs measures for 2D NLS and 3D Hartree NLS equations using the method of random averaging operators, a first order approximation to the full random tensor theory. In this talk we will describe these results, and explain the ideas behind them. We conclude with some open problems and within this context we'll briefly discuss recent joint with Bjoern Bringmann, Yu Deng and Haitian Yue establishing the invariance of the 3D Gibbs measure under the flow of the nonlinear wave equation.

Mamoru Okamoto, Osaka University
Title: Stochastic quantization of the Φ^3_3 model

Abstract: We study the construction of the Φ^3_3-measure and the corresponding dynamical problem. We prove the normalizability of the Φ^3_3-measure, which is singular with respect to the massive Gaussian free field. We also consider the dynamical problem for the canonical stochastic quantization of the Φ^3_3-measure, namely, the three-dimensional stochastic damped nonlinear wave equation with a quadratic nonlinearity forced by an additive space-time white noise. This is a joint work with Tadahiro Oh (University of Edinburgh) and Leonardo Tolomeo (University of Bonn).

Itsuko Hashimoto, Kanazawa University
Title: Asymptotic behavior toward radially symmetric stationary solutions of the compressible Navier-Stokes equation

Abstract: This talk is concerned with the asymptotic behaviors of radially symmetric stationary solutions for the compressible Navier-Stokes equation. We discuss the motion of viscous barotropic gas without external forces, where boundary and far field data are prescribed on the exterior domain. For both inflow and outflow problems, the asymptotic behaviors of radially symmetric stationary solutions are shown in a suitably small neighborhood of the initial data. This research is the joint work with Professor Nishibata and Sugisaki of Tokyo Institute of Technology and Matsumura of Osaka university.

Hirotatsu Nagoji, Kyoto University
Title: Renormalization of stochastic nonlinear heat and wave equations driven by subordinate cylindrical Brownian noises

Abstract: In this talk, we consider the stochastic nonlinear heat equations (SNLH) and stochastic nonlinear wave equations (SNLW) on two-dimensional torus driven by a subordinate cylindrical Brownian noise, which we define by the time-derivative of a cylindrical Brownian motion subordinated to a nondecreasing cadlag stochastic process. To construct the solution, we introduce a suitable renormalization. For SNLH, we cannot expect the time-continuity for the solutions because the noise is jump-type. Moreover, due to the low time-integrability of the solutions, I could establish a local well-posedness result for SNLH only with a quadratic nonlinearity. On the other hand, for SNLW, the solutions have time-continuity and we can show the local well-posedness for general polynomial nonlinearities. Through this example, we can see that the heat case behaves worse than the wave case in the singular noise of jump-type cases.

Laurent Thomann, Universite de Lorraine
Title : Almost sure scattering for the one dimensional nonlinear Schrödinger equation

Abstract : We exhibit measures on the space of initial data for which we describe the non trivial evolution by the linear Schrödinger flow and we show that their nonlinear evolution is absolutely continuous with respect to this linear evolution. Actually, we give precise (and optimal) bounds on the Radon-Nikodym derivatives of these measures with respect to each other and we characterise their L^p regularity. We deduce from this precise description the global well-posedness of the equation for $p > 1$ and scattering for $p > 3$. This is joint work with Nicolas Burq.

(HYD2 Joint meeting with RIMS) Gennady EI, Northumbria University
Title: Emergent hydrodynamics of soliton gases in integrable systems

Abstract: Soliton gases represent infinite random ensembles of interacting solitons displaying nontrivial large-scale behaviours ultimately determined by the properties of the elementary two-soliton collisions. The emergent hydrodynamics of non-equilibrium soliton gases in integrable dispersive systems such as the Korteweg-de Vries and nonlinear Schrödinger equations is described by the universal nonlinear integro-differential kinetic equation for the density of states in the spectral (Lax) phase space. In my talk, I will outline the main ideas of the spectral theory of soliton gases and its connection with the fundamental concept of “integrable turbulence” introduced by Zakharov in 2009.

Sandra Cerrai, University of Maryland
Title: On the small mass limit for some stochastic wave equations with variable friction and polynomial nonlinearities.

Abstract: We study the validity of the Smoluchowski-Kramers approximation for a class of stochastic nonlinear damped wave equations, including equations of Klein-Gordon type. We will also study the validity of a large deviation principle, in the joint small mass and small noise limit. The friction term is assumed to be state dependent.

Masaya Maeda, Chiba University
Title: Small energy stabilization of 1D nonlinear Klein-Gordon equation with potential

Abstract: This talk is based on the joint work with S.Cuccagna and S.Scrobogna. We give a partial extension to dimension 1 of the result proved by Bambusi and Cuccagna on the absence of small energy real valued periodic solutions for the nonlinear Klein Gordon equation in dimension 3. We combine the framework in Kowalczyk and Martel with the notion of "refined profile."

Yukimi Goto, Kyushu University
Title: Born-Oppenheimer Potential Energy Surfaces for Kohn-Sham Models in the Local Density Approximation

Abstract : We consider a system of two electrically neutral (equal number of protons and electrons) atoms. In quantum mechanics textbooks, these atoms attract each other with a long-range force called the van der Waals force. This fact has been rigorously proved by Lieb and Thirring in non-relativistic Schroedinger theory (Phys. Rev. A 1986). However, the repulsive force should be stronger for small nuclear separation. In this talk, I will explain that this repulsive force is about the -7th power of the distance R, using a nonlinear equation called the local density approximation based on the density matrix functional theory.

Aurelien Deya, Universite de Lorraine
Title: A few results about the hyperbolic Anderson model

Abstract: We will report on recent results about the following *hyperbolic Anderson model*:

$$\frac{\partial^2 u}{\partial t^2}(t,x) = \Delta u(t,x) + u\,\dot{B}(t,x), \quad t \in [0,T],\ x \in \mathbb{R}^d,\ d \in \{1,2,3\},$$

where $\dot{B}$ is a space-time fractional noise of index $H = (H_0, ..., H_d) \in (0,1)^{d+1}$, that is a Gaussian noise on $\mathbb{R}_+ \times \mathbb{R}^d$ with covariance (formally) given by

$$\mathbb{E}\big[\dot{B}(s,x)\dot{B}(t,y)\big] = |s-t|^{2H_0-2} \prod_{i=1}^{d} |x_i - y_i|^{2H_i-2}.$$

Fractional noises are known to model long-range-dependence phenomena, in contrast with the standard white-noise situation. Modifying the index H also allows us to calibrate the roughness of the noise.

As in the case of the fractional SDE, the above equation can be investigated along two directions:
(1) The *stochastic* approach, based on the Wick product and the related *Skorohod integral.*
(2) The *pathwise* approach, based on the construction of the *Young (wave) integral.*

We will discuss about these two possible interpretations, and account for the corresponding results on wellposedness of the model.

The talk will be based on two joint works with Xia Chen, Jian Song and Samy Tindel.

Anne-Sophie de Suzzoni, Ecole Polytechnique
Title : General remarks on the propagation of chaos in wave turbulence and application to the incompressible Euler dynamics

Abstract : In this talk, we will present a result regarding propagation of chaos in the context of wave turbulence. That is, considering the solution to Hamiltonian PDEs having at initial time independent Gaussians as Fourier coefficients, we prove that at later times, the Fourier coefficients remain asymptotically independent Gaussians in the sense that they satisfy the Wick formula. The Wick formula at later times is asymptotic in the sense that we work on a torus whose size grows to infinity. We then apply this result to the incompressible Euler equation.

(HYD2 Joint meeting with RIMS) Christof Melcher, RWTH Aachen University
Title: Emergent spin-orbit coupling and rotating skyrmions

Abstract: We discuss solitonic field configurations on a spherical magnet. Exploiting the Hamiltonian structure and concepts of angular momentum, we present a new family of localized solutions to the Landau-Lifshitz equation that are topologically distinct from the ground state and break rotational symmetry. The approach illustrates emergent spin-orbit coupling arising from the loss of individual rotational invariance in spin and coordinate space - a common feature of condensed matter systems with topological phases.

Takiko Sasaki, Musashino University/Tohoku University
Title: The combined effect of one space dimension beyond the general theory for nonlinear wave equations

Abstract: In this talk, we introduce lifespan estimates for a special class of semilinear wave equations in one space dimension for which the so-called combined effect is observed. It is remarkable that, including the combined effect case, our results on the lifespan estimates are partially better than those of the general theory for nonlinear wave equations. This is a joint work with Professor Hiroyuki Takamura and Mr. Katsuaki Morisawa.

Minami Watanabe, Tsuda College
Title: Global dynamics of nonlinear Schrödinger equation with double power nonlinearity

Abstract: In this presentation, we consider the global behavior of solutions to the Schrödinger equation (NLS) with double-power nonlinear terms. The Schrödinger equation, which is classified as a nonlinear dispersive equation, is classified into a scattering solution, an explosive solution, and a standing wave solution, depending on the competition between wave dispersiveness and nonlinearity. The goal is to classify the above three solutions by the initial values. Therefore, we set up potential wells for positive and negative virial functionals, and report the results of investigating the solutions of (NLS) starting from each set. We also touch on recent results on the global behavior of solutions to the nonlinear Schrödinger equation.

Svetlana Roudenko, Florida International University
Title: Dynamics of solutions in stochastic nonlinear Schrödinger equation: critical and supercritical cases

Abstract: We consider the one-dimensional focusing stochastic nonlinear Schrödinger (SNLS) equation with linear and nonlinear noise in the L^2-critical and supercritical settings. We look at additive and multiplicative perturbations driven by space-time white noise and multiplicative noise driven by a Wiener process white in time and colored in space. We discuss the influence of noise on conserved (in deterministic setting) quantities, and then on the dynamics of solutions. We discuss various thresholds for solutions behavior and especially investigate the noise influence onto perturbations of solitons. We discuss quantitative estimates and also numerical findings. The talk is based on joint work with Annie Millet, Alex D. Rodriguez and Kai Yang.